

## TEMPORAL CHARACTERISTICS OF THE 2012 PERNIK EARTHQUAKE AFTERSHOCK SEQUENCE

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**Abstract:** Statistical analysis is applied to study temporal pattern of earthquake distribution in aftershock sequence of the 2012 Mw=5.6 Pernik earthquake. On the assumption that aftershocks are distributed in time as a non-stationary Poisson process the maximum likelihood method for estimating the parameters ( $K$ ,  $c$  and  $p$ ) of the modified Omori formula is used. A transformation from the time scale  $t$  to a frequency-linearized time scale  $\tau$  is applied for testing the goodness of fit between the aftershock occurrence and different statistical models. It is found that the temporal distribution of the Pernik earthquake aftershocks is dominated by the classic power law decay in time.

**Key words:** aftershock, aftershock sequence, aftershock decay rate, southern Bulgaria

### Introduction

Study of the space-time distribution of earthquakes is of fundamental importance for understanding the physics of the earthquake generation process. The spatial and temporal clustering of aftershocks is the dominant non-random element of seismicity, so when aftershocks are removed, the remaining activity can be modelled (as first approximation) as a Poisson process (Gardner and Knopoff, 1974). Aftershocks occur after the mainshock and their frequency decays through time with approximately the reciprocal of time elapsed since the main earthquake. The occurrence rate of aftershock sequence in time is empirically well described by the modified Omori formula proposed by Utsu in 1961 (Utsu, 1969). The power-law decay represented by the modified Omori relation is an example of temporal self-similarity of the earthquake source process. Aftershock decay rate (parameter  $p$ ) may contain information about the mechanisms of stress relaxation and frictional strength heterogeneity, (Mikumo and Miyatake, 1979), but this information can not to be derived without precise characterization of the empirical relations that best fit the

data.

In the present study statistical analysis to examine the temporal pattern of earthquakes in the aftershock sequence of the Mw=5.6 Pernik earthquake is applied. The earthquake occurred on May 22, 2012 in Sofia seismic zone south-western Bulgaria. To estimate the parameters ( $K$ ,  $c$  and  $p$ ) in the modified Omori formula the maximum likelihood method for the aftershock sequence is used. The goodness of fit between the aftershock occurrence and different statistical models has then been tested on the base of transformation of the time scale  $t$  to a frequency-linearized time scale  $\tau$ . Further the Akaike Information Criterion (Akaike, 1974) is used to select the best model among competing models.

## Method and Data

### Method

The frequency  $n(t)$  of aftershocks at time  $t$  is well represented by the modified Omori formula (Utsu, 1969):

$$n(t) = K(t+c)^{-p} \quad (1)$$

where  $t$  is the elapsed time since the occurrence of the main shock, and  $K$ ,  $p$ ,  $c$  are constant parameters. The most important parameter  $p$ , characterizes the decay of the aftershock activity.

On the assumption that aftershocks are distributed as a non stationary Poisson process with intensity function  $\lambda(t; \theta)$ ,  $\theta=(K, p, c)$ , Ogata (Ogata, 1983) used the maximum likelihood method to estimate the parameters  $K$ ,  $c$  and  $p$  in modified Omori formula (1).

The intensity function of the Poisson process  $\lambda(t)$  is defined by the relation:

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \text{Prob}\{\text{an event in } [t, t+\Delta t]\} / \Delta t \quad (2)$$

Using the modified Omori formula, the intensity function becomes:

$$\lambda(t, \theta) = K(t+c)^{-p} \quad (3)$$

An integration of the intensity function  $\lambda(t)$  gives a transformation of the time scale  $t$  to a frequency-linearized time scale  $\tau$  (Ogata and Shimazaki, 1984). If the choice of the intensity function  $\lambda(t)$  (i.e., the parameters  $K$ ,  $c$  and  $p$ ) is correct the occurrence of aftershocks becomes the standard stationary Poisson process on the frequency-linearized time axis.

The frequency-linearized time for an aftershock sequence can be defined as:

$$\tau = A(t) = \int_s^t \lambda(s) ds \quad (4)$$

The time scale  $\tau$  is used for testing the goodness of fit between the aftershock occurrence and the selected model. A linear dependence between the observed cumulative number of aftershocks ( $N$ ) and  $\tau$  should be observed if an appropriate model has been selected. Anomalies in the aftershock activity are more evident on the  $N(\tau)$  plot than on  $n(t)$ . Thus the  $\tau$  time axis will be used to detect secondary aftershock activity.

To select which model fits the observations better, the Akaike Information Criterion (AIC) (Akaike, 1974) is used. The AIC is defined by:

$$\text{AIC} = (-2) \text{Max}(\ln\text{-likelihood}) + 2(\text{Number of the used parameters}) \quad (5)$$

A model with a smaller value of AIC is considered to be a better fit to the observations.

## Data

All events that satisfy the criteria introduced by Gardner and Knopoff (Gardner, J.K., L.Knopoff, 1974) and modified by Christoskov and Lazarov (Christoskov, L., R. Lazarov, 1981) for the Central Balkans, accepted as aftershocks

$$\log(R_a) = 0.9696 + 0.1243M$$

$$\log(T_a) = -0.62 + 0.56 M \quad (M < 6.0)$$

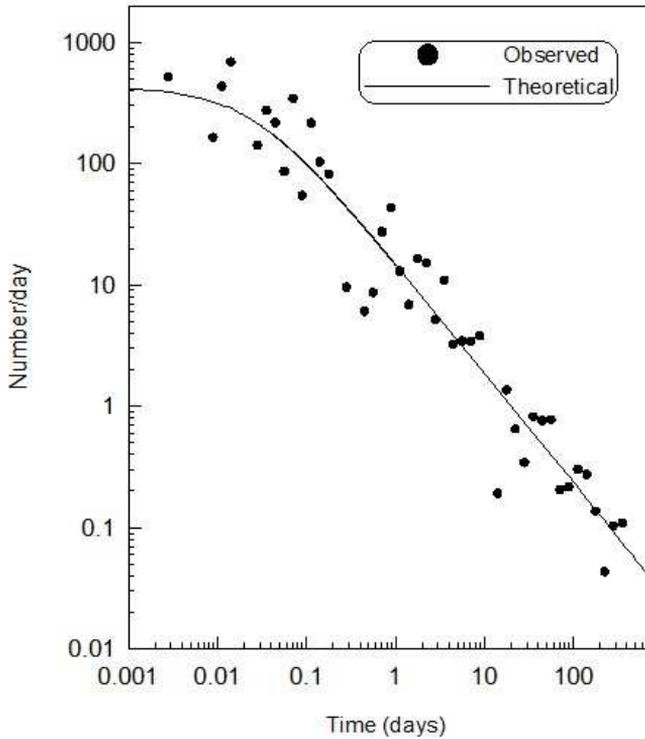
$$\log(T_a) = -5.25 + 2.15M - 0.137M^2 \quad (M \geq 6.0) \quad (6)$$

where  $M$  is surface wave magnitude of the main event,  $R_a$  is the greatest distance between the main event and aftershock, and  $T_a$  is the greatest elapsed time since the occurrence of the main shock.

For the time period May 2012 - May 2013 aftershock sequence of the  $M_w = 5.6$  Pernik earthquake is compiled on the base of digital data from NOTSSI (National Operative Telemetric System for Seismological Information). More than 1000 aftershocks occurred in the considered time interval but only about 20% of them are localized (recorded at least at three stations). The majority of the aftershocks are recorded only at seismic station Vitosha (VTS)-the nearest station (at about 15 km from the main shock epicenter). Thus the analyzed sequence comprises 169 ( $M \geq 1.0$ ) aftershocks occurred in time interval of 365 days.

## Results

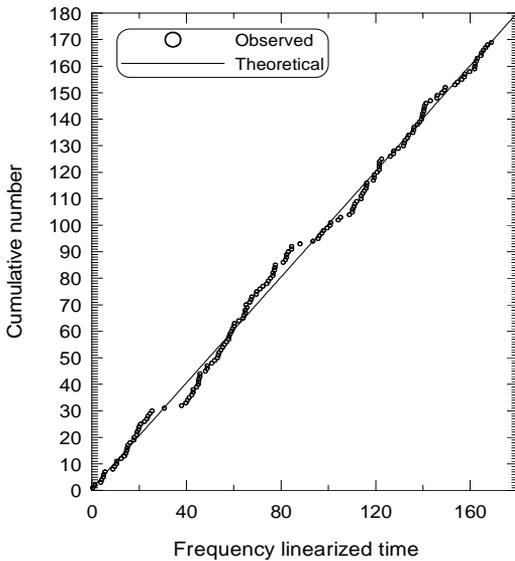
The aftershock sequence from 0 to  $T=365$  days after the  $M_w=5.6$  Pernik earthquake applying to the modified Omori formula (1) was analyzed. The maximum likelihood estimates (MLM's) are as follows:  $K=15.23$ ,  $c=0.025$ ,  $p=0.90$ . The frequency-time distribution of aftershocks is presented in Figure 1.



**Fig. 1.** Frequency - time distribution of aftershocks.

In the figure the observed distribution is compared to the distribution (called theoretical), which is expected as a results from the selected model (in the case, the model is the modified Omori formula). The comparison between empirical and theoretical distribution (Fig.1) shows that as a first approximation the temporal distribution of earthquakes in aftershock sequence of the 2012  $M_w=5.6$  Pernik earthquake is well described by the modified Omori formula.

The cumulative number of events is plotted against the frequency-linearized time  $\tau$  defined in the equation (4) using the MLM's of parameters  $K$ ,  $p$ ,  $c$  (presented in Fig. 2). A relatively good agreement between the expected (theoretical) and the observed distribution of the aftershocks can be seen in Figure 2. The figure shows that a nearly linear trend of aftershock decay continues up to 365 days (177 linearized time); thus the modified Omori formula (1) fits largely the observations up to 365 days after the main shock.



**Fig. 2.** Plot of cumulative number of events versus frequency linearized time  $\tau$ .

The figure also suggests the existence of some discrepancies between the observed and expected distributions (S-shaped deviations from the linear trend) – evident periods of decaying and activation of the process. An increase of the cumulative number is observed in the first hours after the main shock (time period of twelve hours 36 aftershocks occurred, among them the strongest one) followed by a decrease in number of events several hours later. The aftershock activity decreases before the 40<sup>th</sup> day and increases about 10 to 20 days later. This temporal distribution anomaly could be caused by the occurrence of one of the strongest aftershock that occurred 53 days (on July 14, 2012 with  $M=4.4$ ) after the mainshock.

Consequently two models that take into account the effect of secondary aftershock activity were constructed: 1) the first model with one secondary aftershock sequence after 0.5 days and 2) a combination of one main and two secondary sequences - after 0.5 and 53 days. The same  $p$  value for the main and secondary aftershock sequences is assumed for both models.

The maximum likelihood estimate of the parameters in the modified Omori formula, and the selection of a statistical model based on AIC, show that the aftershock sequence of the 2012 Pernik earthquake is best modeled by one ordinary and two secondary sequences, although there remain S-shaped deviation from the linear trend -about 80 day after the main shock.

## **Discussion and conclusions**

The aftershock sequence from 0 to 365 days after the 2012 Pernik earthquake (occurred in Sofia seismic zone – south-western Bulgaria) by fitting it to the modified Omori formula was analyzed. The aftershock sequence of the 2012 earthquake (its

magnitude estimates according to USGS/NEIC are:  $M_w=5.6$ ;  $m_b=5.7$ ;  $M_s=5.2$ ) is compiled on the base of digital data from NOTSSI. For homogeneity a threshold magnitude (the minimum magnitude of aftershocks) of  $M=1.0$  is accepted. Thus the studied sequence comprises 169 aftershocks occurred in time interval of 365 days.

The  $p$  value ( $p=0.90$ ) estimated for the aftershock sequence is in the middle of  $p$  value range obtained for aftershock sequences in Bulgaria, i.e.,  $p=0.71-1.17$  (Simeonova, Solakov, 1999).

The temporal distribution of the aftershocks following the 2012  $M_w=5.6$  earthquake is similar to the aftershock sequences in northern Bulgaria obtained by Simeonova and Solakov (1999) - slowly decay in time ( $p<1.0$ ), without secondary aftershock activity. By contrast, the sequences in south-western Bulgaria are characterized by well expressed secondary aftershock activity and relatively high values of  $p$  for the main sequence ( $p>1$ ) (Simeonova, Solakov, 1999).

The main conclusions are as follows:

- The maximum likelihood estimate of the parameters in the modified Omori formula and the selection of a statistical model based on AIC show that the aftershock sequence of the 2012 Pernik earthquake is best model by one ordinary and two secondary sequences.
- The temporal pattern of the earthquakes distribution in aftershock sequence of the 2012  $M_w=5.6$  Pernik quake (occur in south-western Bulgaria) is similar to the temporal distribution of the aftershocks in Northern Bulgaria - slow decay in time ( $p<1.0$ ), without secondary aftershock activity.
- It can be assumed that aftershocks are generated in slowly relaxing environment with low heterogeneity.

## References

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## **Времени характеристики на афгършоковата активност след земетресението от 2012 около град Перник**

Пл.Райкова

**Резюме.** Статистическият анализ се прилага за изучаване времеви модел на афгършоковата активност последвала след земетресението през 2012  $M_w = 5.6$  в района на град Перник. Въз основа на предположението, че вторичните трусове са разпределени във времето като нестационарен Пуасонов процес се използва модифицираната формула на Омори за оценяване на параметрите ( $K$ ,  $c$  и  $p$ ). Трансформацията на времето скала  $t$  в честота-линеализирана скала  $\tau$  се прилага за определяне на най-добрата връзка между афгършоковата активност и различни статистически модели.